

Ex.19 证明. 因为

$$\text{rank}(PA) \leq \min(\text{rank}(P), \text{rank}(A)) \leq \text{rank}(A).$$

另一方面,

$$\text{rank}(A) = \text{rank}(P^{-1}PA) \leq \text{rank}(PA).$$

所以, $\text{rank}(A) = \text{rank}(PA)$.

同理可证: $\text{rank}(A) = \text{rank}(AQ)$.